International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 4 Issue 10 ǁ October. 2016 ǁ PP. 13-18

Study for Parameter Estimation Optimization Based on Q-GRBPF

Guan Cheng[1]， Fang Yu[1]， Hu Ding-Yu[1]
(School of Urban Railway Transportation，Shanghai University Of Engineering Science，Shanghai 201620，China)

Abstract: Considering the problem of high computational complexity in the Rao-Blackwellized particle filter for the high performance requirements of parameters estimation algorithm for railway vehicles, a recursive estimation algorithm based on the quasi-Gaussian Rao-Blackwellized particle filter was put forward for the mixed linear and nonlinear models in the parameter estimation. The state space was divided into the linear part and the nonlinear part in the algorithm. The nonlinear part was estimated by the quasi-Gaussian particle filter, the filtering distribution was approximated as a Gaussian distribution, the resampling was omitted, and a search algorithm was adopted in the importance sampling; the linear part was estimate by the Kalman filter with the estimated values of the nonlinear part. The simulation results show that: compared with the Rao-Blackwellized particle filter, the quasi-Gaussian Rao-Blackwellized particle filter effectively reduces the computational complexity, cuts down computation time under ensuring estimate accuracy, and provides the theoretical guidance for the study on parameter estimation algorithm of the railway vehicles.
Keywords: Railway vehicle; Quasi-Gaussian Rao-Blackwellized particle filter; Rao-Blackwellized particle filter; Parameter estimation

I. INTRODUCTION

In recent years, the performance parameter estimation of rail vehicles has become the research focus of rail vehicle on-line monitoring, mainly because the evolution of vehicle service performance is reflected by the parameter change, and only a small amount of sensors can be used for safety monitoring of the state of vehicle suspension system in actual application.

Because particle filter (PF) can deal with non-linear and non-Gaussian problems in parameter estimation, domestic and foreign scholars have done a lot of research on particle filter and its improved algorithm [1-3]. At present, the more mature particle filter technology is the edge particle filter (Rao-Blackwellized particle filter, RBPF). In the literature [4], the RBPF is combined with the dynamics model of the rail vehicle, the probabilistic model of the performance parameter estimation of the rail vehicles is divided into blocks, and Kalman filter (KF) is used to predict and measure the state block, and the particle filter (PF) is used to predict and update the parameter block, which improves the estimation precision and the convergence of the algorithm. In this paper, the particle filter (PF) is used to reduce the state space of the particle filter. However, when RBPF is used to estimate the nonlinear state, the classical particle filter is also used, so the computational complexity of the particle filter is high (the most part is from the resampling strategy in the algorithm), which limits its robustness to a certain extent. Practical application. In order to reduce the computational complexity of resampling in particle filter, the quasi-Gaussian particle filter (Q-GPF) is proposed in which the filter distribution is approximated as a Gaussian distribution and the one-step prediction distributions need not be approximated as a Gaussian distribution [5].

In this paper, the Q-GPF and RBPF are merged, and the parameter estimation model based on the quasi-Gaussian Rao-Blackwellized particle filter (Q-GRBPF) algorithm is proposed according to the literature [5]. The filtering distribution is approximated as the Gaussian distribution, and the search algorithm is used in the importance sampling. Then, the effectiveness of the Q-GRBPF algorithm is verified by an example.

II. THEORY OF Q-GRBPF ALGORITHM

Assuming that the system is divided into linear and nonlinear parts, the non-linear state-space system model can be expressed as follows:

$$x_{t+1}^n = f(x_t^n) + A_t^n x_t^l + B_t^n w_t^n \quad (1a)$$

$$x_{t+1}^l = A_t^l x_t^l + B_t^l w_t^l \quad (1b)$$

$$y_t = h(x_t^n) + e_t \quad (1c)$$

In the above equations, the x_t^n and x_t^l are the nonlinear state and the linear state at time t respectively, and $[x_t^n, x_t^l] = x_t$, $x_t \in R^{n_x}$,（$t \in N$, n_x is the system state dimension and R^{n_x} is the state space）; the observation sequence is $y_t \in R^{n_y}$ （n_y denotes the system observation dimension); w_t^n and w_t^l , respectively, denote the zero mean gaussian white noise sequence of the non-linear state and the linear state, respectively; $[w_t^n, w_t^l] = w_t$, and its distribution is known (Q is its variance), which is independent of the system state; e_t is the observed noise sequence, the distribution is known (R is its variance) and is not related to the system state and state noise; $f(\bullet)$ and $h(\bullet)$ are the known functions. The filtered distribution can be obtained by Bayes principle

$$p(x_t \mid y_{0:t}) = p(x_t^l, x_t^n \mid y_{0:t}) = p(x_t^l \mid x_t^n, y_{0:t}) p(x_t^n \mid y_{0:t}) \quad (2)$$

The $p(x_t^l \mid x_t^n, y_{0:t})$ is assumed to be a linear Gaussian system, so the Kalman filter can be used to obtain the optimal estimation in the sense of minimum variance, but $p(x_t^n \mid y_{0:t})$ is the nonlinear system, which is estimated by the quasi-Gaussian particle filter.

III. PARAMETER ESTIMATION THEORY AND MODEL BASED ON Q-GRBPF

The $q(x_t \mid y_{1:t}) = p(x_t \mid y_{1:t-1})$ is adopted in the parameter estimation model in this paper[5]. Assuming the system initial state estimation is x_0, the system initial state estimation variance P_0 is a 2-order matrix, the variance P_0 generally takes a small value, the range of the unknown parameter vector θ is $[\theta_{\min}, \theta_{\max}]$, and the number of the particles is N. Therefore, the steps for the parameter estimation of the Q-GRBPF algorithm are summarized below:

Step 1: Initialize phase

For $i = 1, \cdots, N$, in the interval $[\theta_{\min}, \theta_{\max}]$, sample uniformly, constitute the particle set $\boldsymbol{\theta}_{1|0}(i)$, the system initial state is $\boldsymbol{x}_{1|0}(i)=\boldsymbol{x}_0$, and the system initial variance is $\boldsymbol{P}_{1|0}(i)= \boldsymbol{P}_0$.

Step 2: Parameter estimation phase. At each time k (k = 1, 2, ...), the following steps are repeated.

(1) For $i = 1, \cdots, N$, calculate the weight $\tilde{w}_k(i)$ of each particle, and its corresponding normalized weight $w_k(i)$

$$y_{k|k-1}(i) = H(\theta_{k|k-1}(i)) x_{k|k-1}(i) \quad (3)$$

$$R_k(i) = H(\theta_{k|k-1}(i)) P_{k|k-1}(i) H^T(\theta_{k|k-1}(i)) + R \quad (4)$$

$$\tilde{w}_k(i) \sim N(y_{k|k-1}, R_k(i)) \quad (5)$$

$$w_k(i) = \frac{\tilde{w}_k(i)}{\sum_{j=1}^{j=N} \tilde{w}_k(j)} \quad (6)$$

(2) Calculate the parameter mean and its variance. The estimated value $\hat{\theta}_k$ of the parameter at time k is

$$\hat{\theta}_k = \sum_{i=1}^{i=N} w_k(i) \theta_{k|k-1}(i) \quad (7)$$

$$P_k = \sum_{i=1}^{i=N} (\theta_{k|k-1}(i) - \hat{\theta}_k)^T (\theta_{k|k-1}(i) - \hat{\theta}_k) w_k(i) \quad (8)$$

(3) particle filter measurement update. The particles are available from the filtering distribution at time k

$$\theta_k(i) \sim p(\theta_k \mid y_{0:k}) \approx N(\hat{\theta}_k, P_k) \quad (9)$$

Now the weight and its normalized weight of each particle are

$$\tilde{q}_k(i) \subset N(\theta_k(i), P_k(i)) \quad (10)$$

$$q_k(i)=\frac{\tilde{q}_k(i)}{\sum_{i=1}^{i=N}\tilde{q}_k(i)} \quad (11)$$

(4) Kalman filter measurement update.

$$\boldsymbol{R}_k(i)=\boldsymbol{H}(\boldsymbol{\theta}_k(i))\boldsymbol{P}_{k|k-1}(i)\boldsymbol{H}^{\mathrm{T}}(\boldsymbol{\theta}_k(i))+\boldsymbol{R} \quad (12)$$

$$\boldsymbol{K}_k(i)=\boldsymbol{P}_{k|k-1}(i)\boldsymbol{H}^{\mathrm{T}}(\boldsymbol{\theta}_k(i))\boldsymbol{R}_k^{-1} \quad (13)$$

$$\boldsymbol{x}_k(i)=\boldsymbol{x}_{k|k-1}(i)+\boldsymbol{K}_k(i)(\boldsymbol{y}_k-\boldsymbol{H}(\boldsymbol{\theta}_k(i))\boldsymbol{x}_{k|k-1}(i)) \quad (14)$$

$$\boldsymbol{P}_k(i)=\boldsymbol{P}_{k|k-1}(i)-\boldsymbol{K}_k(i)\boldsymbol{H}(\boldsymbol{\theta}_k(i))\boldsymbol{P}_{k|k-1}(i) \quad (15)$$

(5) particle update. The particles are resampled from the importance distribution function generated by Q-GRBPF

$$\theta_{k+1|k}(i)\sim q(\theta_{k+1}\mid y_{0:k}) \quad (16)$$

M random numbers u_p(p=1，2，…，M) are generated ,which are uniformly distributed in the interval [0,1], and then the integer l is found by the search algorithm that satisfies the following conditions:

$$\sum_{j=1}^{j=l-1}q_k(j)<u_p\le\sum_{j=1}^{j=l}q_k(j) \quad (17)$$

In the above inequality, l = 1, 2, 3, ..., N , the lth particle is recorded, which is resampled by N times, and obtain the new parameters, state and variance combination $\boldsymbol{\theta}_{k+1|k}(l)$，$\boldsymbol{x}_k(l)$，$\boldsymbol{P}_k(l)$.

(6) Kalman filter step prediction update.

$$\boldsymbol{x}_{k+1|k}(i)=\boldsymbol{A}(\boldsymbol{\theta}_{k+1|k}(i))\boldsymbol{x}_k(i) \quad (18)$$

$$\boldsymbol{P}_{k+1|k}(i)=\boldsymbol{A}(\boldsymbol{\theta}_{k+1|k}(i))\boldsymbol{P}_k(i)\boldsymbol{A}^{\mathrm{T}}(\boldsymbol{\theta}_{k+1|k}(i))+\mathbf{B}(\theta_{k+1|k}(i))\mathbf{Q}\mathbf{B}^{\mathrm{T}}(\theta_{k+1|k}(i)) \quad (19)$$

IV. VERIFICATION AND COMPARATIVE STUDY

As a special form of classical particle filter algorithm, Q-GRBPF is applied to parameter estimation. The first problem to be solved is to improve the ability of particle filter to estimate the state, and the performance improvement of the algorithm mainly includes two aspects, that is, improving the efficiency of resampling and choosing the proper importance distribution function. In this paper, the Q-GRBPF algorithm is improved in both aspects: in the resampling section, the posterior probability density is approximated as Gaussian distribution, which greatly reduces the computational complexity compared with the resampling rule in RBPF; in the session of importance sampling, the weight threshold is set to avoid the larger error that generate by the small weight or irrelevant particles.

In order to compare the estimated performance of parameters estimation algorithm of Q-GRBPF and the more mature RBPF, the state space equations of Eq. (20) and Eq. (21) are used in the simulation.

Equation of state: $$x_k=\begin{bmatrix}0 & 1\\ a & b\end{bmatrix}x_{k-1}+\begin{bmatrix}-1 & 0.8\\ -1.3 & 1\end{bmatrix}w_{k-1} \quad (20)$$

Observational equation: $$y_k=\begin{bmatrix}1 & -0.8\\ -0.5 & 0.7\end{bmatrix}x_k+e_k \quad (21)$$

The state noise w_{k-1} and observation noise e_k of the system are Gaussian white noise. The estimated parameters are $\theta=[a,b]^T$, a = -0.7, b = 1.5. The Q-GRBPF and RBPF are used to estimate the parameters a, b.

In the simulation, we choose $Q=diag([0.001,0.001])$， $R=diag([0.001,0.001])$, the initial state $x_0=[0,0]$, the initial variance $P_{0|0}=diag([0.05^2,0.05^2])$, the number of particles N = 1000. $\hat{\boldsymbol{\theta}}_k$ is the estimated value of the model parameters. Based on the comparison between the estimated value and the true value, the rationality of the algorithm is verified.

The simulation results of the algorithm for the parameters a and b are as shown in Figure 1 and 2, respectively. The error estimates for the parameters a and b of the two algorithms are as shown in Figure 3 and 4, respectively. The comparisons for average computation time of the convergence are as shown in Figure 5.

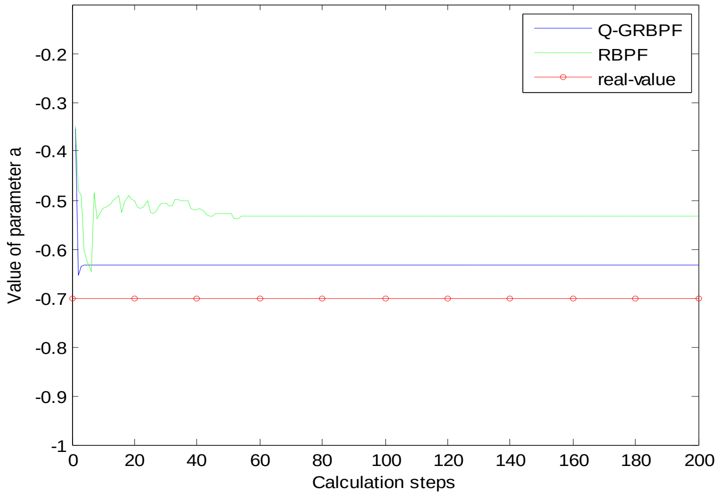

Fig.1 Estimated value of the parameter a

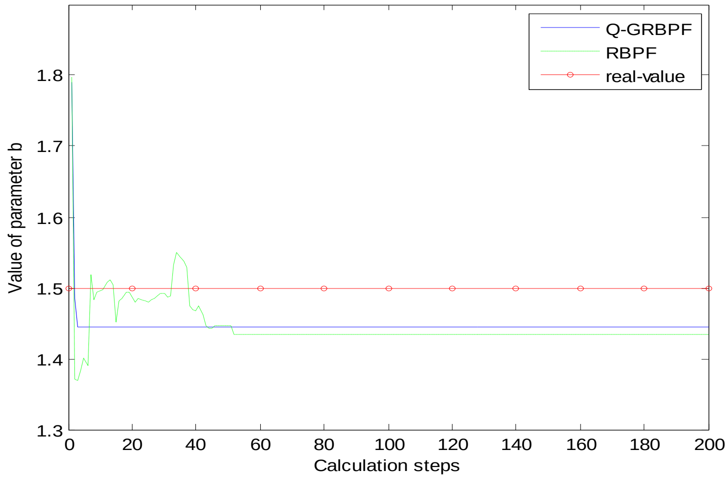

Fig.2 Estimated value of the parameter b

The simulation results in Fig. 1 and Fig. 2 show that both the Q-GRBPF algorithm and the RBPF algorithm converge and obtain satisfactory results after a certain number of iterations.

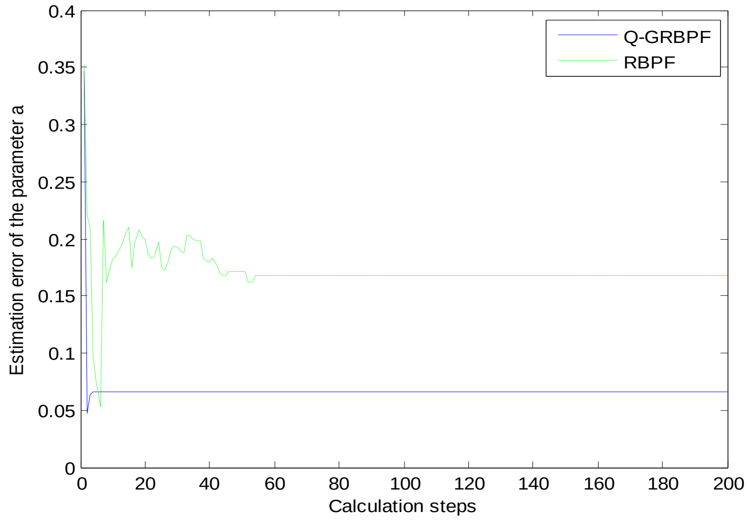

Fig.3 Estimation error of the parameter a

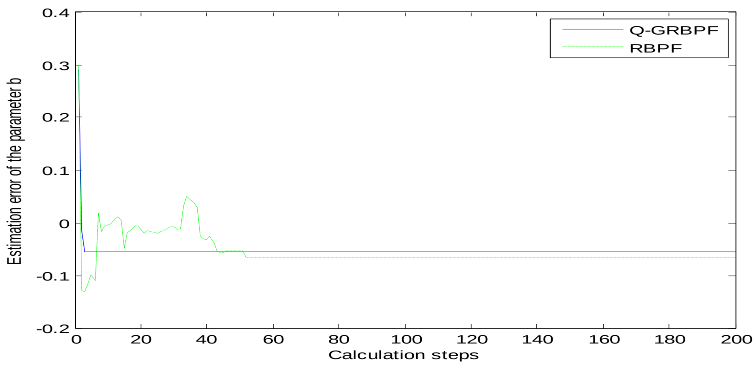

Fig.4 Estimation error of the parameter b

The simulation results in Figure 3 show that the Q-GRBPF algorithm for the identification of the parameter a converges at step 4 with an estimated error of 0.066 and the RBPF algorithm converges at step 55 with an estimated error of 0.167. The estimated accuracy of the Q-GRBPF algorithm is 14.4% higher than RBPF.

The simulation results in Figure 4 show that the Q-GRBPF algorithm converges at step 4 with an estimated error of -0.055 and the RBPF algorithm converges at step 53 with an estimated error of -0.067. The estimated accuracy of the Q-GRBPF algorithm is 0.8% higher than the RBPF algorithm.

Therefore, the Q-GRBPF algorithm outperforms the RBPF algorithm in estimating the parameters within the rational range.

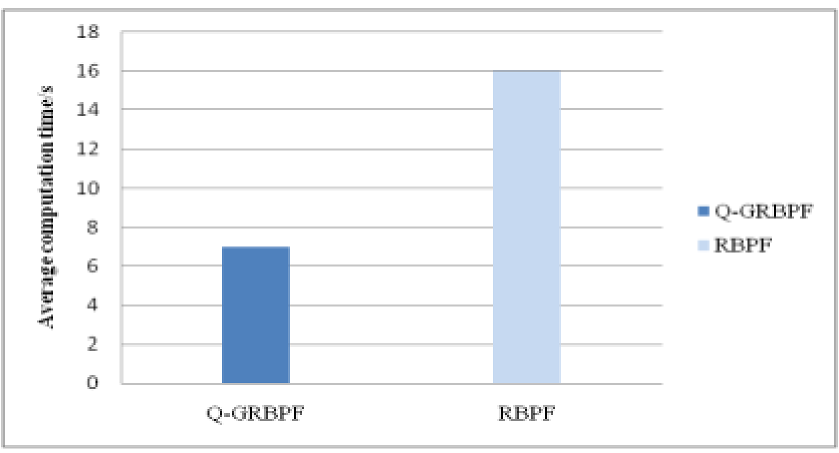

Fig.5 Average computation time of Q-GRBPF and RBPF algorithm convergence

It can be concluded from Fig.5 that the computation time of Q-GRBPF algorithm is about 56% less than that of RBPF algorithm, which is due to the decrease of Q-GRBPF computation complexity. In some applications that request high real-time performance, Q-GRBPF could meet the requirements of real-time filtering.

V. CONCLUSIONS

In this paper, the recursive estimation algorithm based on Q-GRBPF is proposed for mixed linear and nonlinear models in the parameter estimation. This algorithm is a special form of particle filter that improves in both the importance distribution function and resampling. The nonlinear part is estimated by the Q-GPF and the linear part is estimated by the KF in the algorithm. The analysis and simulation results show that compared with RBPF, Q-GRBPF can reduce the computational complexity and has a better convergence property under the premise of guaranteeing the accuracy of estimation. It is more suitable for estimating the suspension parameters of rail vehicles.

REFERENCE

[1]. Li P， Goodall R， Weston P， et al . Estimation of railway vehicle suspension parameters for condition monitoring [J]. Control Engineering Practice， 2007， 15(1)：43-55.

[2]. Fang Yu，Chen Long，Li Xiao-bin，et al. Estimation method for the secondary suspension system parameters of railway vehicle improved by repeated uniform sampling strategy [J]. China Railway Science，2013，34(3)： 72-78.

[3]. Chen Long. THE STUDY ON CONDITION MONITORING METHOD OF URBAN RAIL VEHICLE BASED ON PARTICLE FILTER [D]. Shanghai: Shanghai University of Engineering Science , 2013

[4]. Fang Yu, Chen Long, Zhen Shu-bin, et al. Condition Monitoring of rail vehicle suspension system based on parameter estimation [J]. Journal of the China Railway Society，2013， 35(5)： 15-20.

[5]. Zhuang Ze-sen，Zhang Jian-qiu，Yin Jian-jun. Quasi-Gaussian Rao-Blackwellized Particle Filter for Mixed Linear/Nonlinear State Space Models [J]. Acta Aeronautica et Astronautica Sinica，2008，29（2）：450-455.